LIFE INSURANCE

PREMIUMS AND RESERVES.

BY

SHEPPARD HOMANS,

CONSULTING ACTUARY.

1888.

THE SPECTATOR COMPANY,

16 DEY STREET, NEW YORK.

Life Insurance Premiums and Reserves.

BY SHEPPARD HOMANS, CONSULTING ACTUARY.

The basis of every sound system of life insurance is the Mortality Table. While nothing is more uncertain than the duration of an individual life, the rates of mortality, or, in other words, the probabilities of living and dying in any one year at each age among a large number of persons similiarly situated as regards family history, climatic influences, etc., can be predicted with almost mathematical precision. The rates of mortality among insured lives at the several ages have been carefully ascertained by observations among a vast number of persons insured in British and American companies. These results are embodied in three mortality tables of standard authority, viz:

The Actuaries, or Combined Experience Table, deduced from the mortuary statistics of seventeen British companies, and published in 1837.

The New Actuaries or Hm. Table, deduced from the later experience of twenty British companies, and published in 1869.

The American Experience Table, deduced chiefly from the mortuary statistics of the Mutual Life Insurance Company of New York.

Of these the last named table, confirmed, as it has been in a remarkable degree, by the experience of other American companies, is by far the best index of the rates of mortality which may be expected to prevail among insured lives in the United States. This table has been adopted by nearly all American companies as a basis for premiums and reserves, and by many States as a standard of valuation for contingent insurance liabilities.

These tables do not differ materially from each other, and either would be a safe basis for the transactions of American life insurance companies. Their teachings have all the force of natural laws, and these teachings cannot be disregarded or violated with impunity.

Columns (1) and (2) of the following Table No. 1, show respectively the numbers living and dying at each successive age out of 100,000 persons starting at the age of ten years. Column (3) shows for each age the rate of mortality, or probability of dying within one year. This is also the cost, without interest, to insure one dollar, or unity, payable in case of death within the year, and is found, for any age, by dividing the number of deaths by the number living. For instance, at age 40, dividing 765, the number dying, by 78,106, the number living, we have .009794 as the

Table No. 1.

Age. x	Number Living at Each Age. l_x	Number Dying at Each Age. d_x	Probability of Dying at Each Age, Which is Also the Cost to Insure $1.00 for One Year, at Each Age. $\frac{d_x}{l_x}$	Probability of Living Through the Year at Each Age. $1-\frac{d_x}{l_x}$	Cost to Insure $1,000 Payable in case of Death. Am. Exp. 4%. For One Year Only, at Age x	Cost to Insure $1,000 Payable in case of Death. Am. Exp. 4%. Equal Yearly Premiums During Remainder of Life.
	(1)	(2)	(3)	(4	(5)	(6)
10	100,000	749	.007490	.992510	7 20	10.53
11	99,250	746	.007516	.992484	7.23	10.70
12	98,505	743	.007543	.992457	7.25	10.88
13	97,762	740	.007569	.992431	7.28	11.06
14	97,022	737	.007596	.992404	7.30	11.26
15	96,285	735	.007634	.992366	7.34	11.47
16	95,550	732	.007661	.992339	7.37	11.69
17	94,818	729	.007688	.992312	7.39	11.91
18	94,089	727	.007727	.992273	7.43	12.15
19	93,362	725	.007765	.992235	7.47	12.40
20	92,637	723	.007805	.992195	7.51	12.67
21	91,914	722	.007855	.992145	7 55	12 95
22	91,192	721	.007906	.992094	7.60	13.24
23	90 471	720	.007958	.992042	7.65	13.55
24	89,751	719	.008011	.991989	7.70	13.87
25	89,032	718	.008065	.991935	7.75	14.21
26	88,314	718	.008130	.991870	7.82	14.57
27	87,596	718	.008197	.991803	7.88	14.95
28	86,878	718	.008264	.991736	7.95	15.35
29	86,160	719	.008345	.991655	8.02	15.77
30	85,441	720	.008427	.991573	8.10	16.21
31	84,721	721	.008510	.991490	8.18	16.68
32	84,000	723	.008607	.991393	8.28	17.18
33	83,277	726	.008718	.991282	8 38	17 70
34	82,551	729	.008831	.991169	8.49	18.26
35	81,822	732	.008946	.991054	8.60	18 84
36	81,090	737	.009089	.990911	8 74	19 46
37	80,353	742	.009234	.990766	8 88	20.12
38	79,611	749	.009408	.990592	9 05	20.82
39	78,862	756	.009586	.990414	9.22	21.57
40	78,106	765	.009794	.990206	9.42	22.35
41	77,341	774	.010008	.989992	9 62	23.19
42	76,567	785	.010252	.989748	9.86	24.08
43	75,782	797	.010517	.989483	10.11	25.03
44	74,985	812	.010829	.989171	10.41	26.04
45	74,173	828	.011163	.988837	10.73	27.12
46	73,345	848	.011562	.988438	11.12	28.27
47	72,497	870	.012000	.988000	11.54	29.50
48	71,627	896	.012509	.987491	12 03	30.81
49	70,731	927	.013106	.986894	12.60	32.21
50	69,804	962	.013781	.986219	13.25	33.70
51	68,842	001	.014541	.985459	13.98	35.29
52	67,841	1,044	.015389	.984611	14 80	36.98
53	66,797	1,091	.016333	.983667	15.71	38.79
54	65,706	1,143	.017396	.982604	16 73	40.73
55	64,563	1,199	.018571	.981429	17.86	42 79
56	63 364	1,260	.019885	.980115	19.12	45.00
57	62,104	1,325	.021335	.978665	20.52	47 35
58	60,779	1,394	.022936	.977064	22.00	49 87
59	59,385	1,468	.024720	.975280	23 77	52.57
60	57,917	1,546	.026693	.973307	25 67	55.45
61	56,371	1,628	.028880	.971120	27.77	58.54
62	54,743	1,713	.031292	.968708	30.09	61.84
63	53,030	1,800	.033943	.966057	31.90	65 39
64	51,230	1,889	.036873	.963127	35 45	69.18
65	49,341	1,980	.040129	.959871	38 59	73.25
66	47,361	2,070	.043707	.956293	42.03	77.61
67	45,291	2,158	.047647	.952353	45.82	82.28
68	43,133	2,243	.052002	.947998	50.00	87.29
69	40,890	2,321	.056762	.943238	54.58	92.65
70	38,569	2,391	.061993	.938007	59.61	98 39
71	36,178	2.448	.067665	.932335	65.06	104.54

TABLE No. 1—*Continued.*

AGE. x	Number Living at Each Age. l_x	Number Dying at Each Age. dx	Probability of Dying at Each Age, Which is Also the Cost to Insure $1.00 for One Year, at Each Age. $\frac{dx}{l_x}$	Probability of Living Through the Year at Each Age. $1-\frac{dx}{l_x}$	Cost to Insure $1,000 Payable in case of Death. Am. Exp. 4%. For One Year Only, at Age x	Equal Yearly Premiums During Remainder of Life.
	(1)	(2)	(3)	(4)	(5)	(6)
72	33,730	2,487	.073733	.926267	70.90	111.13
73	31,243	2,505	.080178	.919822	77.09	118.21
74	28,738	2,501	.087028	.912972	83.68	125.85
75	26,237	2,476	.094371	.905629	90.74	134.14
76	23,761	2,431	.102311	.897689	98 38	143.19
77	21,330	2,369	.111064	.888936	106.79	153.14
78	18,961	2,291	.120827	.879173	116.18	164.12
79	16,670	2,196	.131734	.868266	126.67	176.30
80	14,474	2,091	.144466	.855534	138.91	189.87
81	12,383	1,964	.158605	.841395	152.50	204.95
82	10,419	1,816	.174297	.825703	167.59	221.82
83	8,603	1,648	.191561	.808439	184.19	240.90
84	6,955	1,470	.211359	.788641	203 23	262.89
85	5,485	1,292	.235552	.764448	226.49	288.62
86	4,193	1,114	.265681	.734319	255.46	318.82
87	3,079	933	.303020	.696980	291.37	354.03
88	2,146	744	.346692	.653308	334.13	394.52
89	1,402	555	.395863	.604137	380.64	441.22
90	847	385	.454545	.545455	437 06	497.08
91	462	246	.532466	.467534	511 99	566.28
92	216	137	.634259	.365741	609.87	649.34
93	79	58	.734177	.255823	705.94	736.31
94	21	18	.857143	.142857	824.18	840.77
95	3	3	1.000000	0.000000	961 54	961.54

rate of mortality or probability of dying within one year, at that age. Column (4) gives for each age the probability of surviving through one year. This is also the cost, without interest, to provide one dollar, or unity, at the end of one year, payable in case of surviving to the end of the year. This is found by dividing the number living at the next higher age, or one year older, by the number living at the age indicated. Thus for age 40, the probability of surviving through one year is found by dividing 77,341, the number living at age 41, by 78,106, the number living at age 40, and is represented by the fraction .990206. This also is the value, without interest, of one dollar, or unity, payable in case a person now aged 40 is alive at the end of one year.

As it is certain that every individual will be either alive or dead at the end of the year, the probabilities of dying and of living in one year at age 40 may be represented as follows:

Probability of dying in one year	.009794
Probability of living through one year	.990206
Certainty of living or dying in one year	1.000000

Column (5) gives the cost, in advance, for each age to secure $1000 payable at the end of the year in case of death within the year, assuming interest at four per cent

per annum. Thus, for age 40, the sum of $9.42 paid in advance is the net cost to secure $1000 payable at the end of the year provided death should occur within the year. Similiarly at age 50, the cost to insure $1000 for one year is $13.25. At age 60, $25.67; at age 70, $59.61, etc. This cost of insurance for one year is, of course, independent of the form of policy contract, or of the age at which the policy was issued, and in general increases each year as a man grows older. These yearly increasing costs of insurance are called natural premiums.

It may be laid down as a fundamental principle that every life insurance company must collect each year, in some way, either by direct payments, or partly from an accumulated fund and partly by direct payments, the cost, according to these natural premiums, to cover the insurance for the year of the net amount at risk on each and every policy in force, based upon the actual age attained, regardless of the age at entry, the form of policy contract, or the scale of premium payments.

These natural premiums, or cost of insurance for each separate year, constitute the basis of all sound life insurance. Theoretically, the receipt each year of the natural premium, or yearly cost of insuring the net amount at risk, based always upon the actual age attained, will enable any company to meet all its insurance obligations at maturity, on each and every policy in force. Practically, it is necessary to add, under any form of policy contract, a margin for necessary expenses, and a further margin to guard against adverse contingencies, such as epidemics, undue withdrawal of sound lives, etc. But it cannot be too clearly stated that natural premium payments, properly loaded, are not only sufficient, but are all-sufficient to meet all the insurance obligations of any company, no matter what may be the forms of its policy contracts or the methods of its premium adjustments. In fact, any payment in excess of the natural premium applied to the net amount at risk and to the actual age attained is outside of, and independent of, insurance, and should go to expenses, contingent fund, investment or surplus. The natural premium in any year pays for the entire insurance during that year, under any and every form of policy contract in any and every company.

Column (6) gives for each age the level or uniform premiums, to continue unchanged through the remainder of life, as the consideration for securing $1000 payable at the end of the year when death occurs. For instance, at age 40 the payment of $22.35 annually in advance is the net premium at that age to secure $1000, payable at the end of the year when death occurs. These level premiums are the commuted equivalents of the natural, or increasing premiums, as shown in column (5).

We will now examine the principles upon which these level premiums are determined.

The first step is to ascertain the net single premium or amount to be paid down in one sum to secure $1000 payable at death, whenever that event shall happen. It is manifest that this single premium is the sum total of the separate costs of insuring one dollar, or unity, in each successive year, discounted at the rate of interest assumed to the present date or age. As we have seen, the net cost without interest at age 40

Age. $40 + n$	Probability that a Person now Aged 40 will Die During Year of Age $40 + n$. $\frac{d_{x+n}}{l_{40}}$	Present Value of $1.00, Payable *Certain*, at the end of n Years. v^{n+1}	Present Value of $1.00, Payable in Case a Man now Aged 40 Dies at the Age of $40 + n$ Years. $\frac{d_{40+n}}{l_{40}} \times v^{n+1}$	Probability that a Man now Aged 40 Years will be Alive at the Beginning of Age $40 + n$. $\frac{l_{40+n}}{l_{40}}$	Present Value of $1.00, Payable *Certain* n Years from Date. v^{n}	Present Value of $1.00, Payable in n Years, Provided a Man now Aged 40 Years be then Alive. $\frac{l_{40+n}}{l_{40}} \times v^{n}$	n
	(1)	(2)	(3)	(4)	(5)	(6)	
40	.009794	.961538	.0094177	1.000000 ×	1.000000	1.00000 =	0
41	.009910	.924556	.0091620	.990206	.961538	.95212	1
42	.010050	.888996	.0089348	.980296	.924556	.90634	2
43	.010204	.854804	.0087225	.970246	.888996	.86254	3
44	.010396	.821927	.0085449	.960042	.854804	.82065	4
45	.010601	.790315	.0083781	.949645	.821927	.78054	5
46	.010857	.759918	.0082505	.939045	.790315	.74214	6
47	.011139	.730690	.0081389	.928187	.759918	.70535	7
48	.011471	.702587	.0080598	.917049	.730690	.67008	8
49	.011869	.675564	.0080179	.905577	.702587	.63625	9
50	.012317	.649581	.0080006	.893709	.675564	.60376	10
51	.012816	.624597	.0080048	.881392	.649581	.57254	11
52	.013366	.600574	.0080275	.868576	.624597	.54201	12
53	.013968	.577475	.0080663	.855212	.600574	.51362	13
54	.014634	.555265	.0081257	.841241	.577475	.48580	14
55	.015351	.533908	.0081960	.826606	.555265	.45899	15
56	.016132	.513373	.0082817	.811257	.533908	.43314	16
57	.016964	.493628	.0083740	.795125	.513373	.40820	17
58	.017848	.474642	.0084712	.778160	.493628	.38412	18
59	.018795	.456387	.0085778	.760313	.474642	.36088	19
60	.019794	.438834	.0086861	.741518	.456387	.33842	20
61	.020843	.421955	.0087950	.721724	.438834	.31672	21
62	.021932	.405726	.0088983	.700881	.421955	.29574	22
63	.023046	.390121	.0089906	.678949	.405726	.27547	23
64	.024185	.375117	.0090722	.655904	.390121	.25588	24
65	.025350	.360689	.0091435	.631718	.375117	.23697	25
66	.026502	.346817	.0091915	.606367	.360689	.21871	26
67	.027629	.333477	.0092131	.579866	.346817	.20111	27
68	.028762	.320651	.0092083	.552237	.333477	.18416	28
69	.029716	.308319	.0091620	.523519	.320651	.16787	29
70	.030612	.296460	.0090753	.493803	.308319	.15225	30
71	.031342	.285058	.0089343	.463191	.296460	.13732	31
72	.031841	.274094	.0087275	.431849	.285058	.12310	32
73	.032072	.263552	.0084526	.400008	.274094	.10964	33
74	.032021	.253415	.0081145	.367936	.263552	.09697	34
75	.031701	.243669	.0077244	.335915	.253415	.08513	35
76	.031124	.234297	.0072923	.303515	.243669	.07413	36
77	.030331	.225285	.0068330	.273090	.234297	.06398	37
78	.029332	.216621	.0063539	.242760	.225285	.05469	38
79	.028116	.208289	.0058562	.213428	.216621	.04623	39
80	.026771	.200278	.0053617	.185312	.208289	.03860	40
81	.025145	.192575	.0048424	.158541	.200278	.03175	41
82	.023250	.185168	.0043052	.133396	.192575	.02569	42
83	.021100	.178046	.0037567	.110145	.185168	.02040	43
84	.018821	.171198	.0032221	.089246	.178046	.01585	44
85	.016542	.164614	.0027230	.070225	.171198	.01202	45
86	.014263	.158283	.0022575	.053684	.164614	.00884	46
87	.011946	.152295	.0018180	.039421	.158283	.00624	47
88	.009526	.146341	.0013940	.027476	.152295	.00418	48
89	.007106	.140713	.0009999	.017950	.146341	.00263	49
90	.004929	.135301	.0006669	.010844	.140713	.00153	50
91	.003150	.130097	.0004097	.005915	.135301	.00080	51
92	.001754	.125093	.0002194	.002765	.130097	.00036	52
93	.000743	.120282	.0000893	.001011	.125093	.00013	53
94	.000230	.115656	.0000267	.000269	.120282	.00003	54
95	.000038	.112207	.0000043	.000038	.115656	.00000	55
	.....	.....	.....	.....	.112207	.....	..
Totals..	.....	.....	.3675747	.....	.....	16.44311	..

to secure $1, payable at the end of one year in case of death during the first year, is .009794. To find its net present value, paid down, we must discount this cost for one year at the rate of interest assumed. The present value of one dollar, payable certain at the end of one year, at four per cent interest, is .961538. The net present value of one dollar, or unity, payable at the end of one year in case of death, on the basis of the American Table—four per cent interest—is for age 40 years .009794 × .961538 =.0094177. [See columns (1), (2), and (3), Table No. 2.] In the same way the net present value of one dollar, or unity, payable at the end of two years, provided a person now aged 40 should die in the second year, or between ages 41 and 42, is found by dividing 774, the number dying, by 78,106, the number living at age 40, and discounting the quotient for two years. Thus $\frac{774}{78106}$ =.009910; this multiplied by 924 556 =.0091620, and this is the cost at age 40 to secure one dollar, or unity, payable at the end of two years in case of death during the second year. Again, the net present value of one dollar, payable in case a man now aged 40 years should he die in the eleventh year, or between ages 50 and 51, is .0080006. These separate values are shown in column No. 3 in Table No. 2. Their sum total is .3675747, and this is the net single premium paid down to secure one dollar, or unity, payable at the end of the year, when a person now aged 40 years dies, whenever that event shall happen.

By a similar course of reasoning the net present value of one dollar, or unity, payable annually in advance during the remainder of life at any age, is the sum total of the present values of the separate chances of surviving during each successive year, discounted to the present date or age. Thus for age 40 the present value of one dollar in advance is unity or one dollar. The present value, without interest, of one dollar, payable in one year, or at age 41, is, as we have seen, .990206. This multiplied by .961538, the discount, gives .95212 as the present value of one dollar, payable at the end of one year, or at age 41, provided a person now aged 40 be then alive. The present value of one dollar, payable in ten years, or at age 50, provided a person now aged 40 be then alive, is $\frac{69804}{78106}$ =.893709 multiplied by .675564 = .60376. These successive net present values are found in column (6). Their sum total is 16.44311, and this is the present value of one dollar per annum in advance during the lifetime of a person now aged 40 years upon the basis adopted.

As already shown, the net single premium at age 40 to secure one dollar, or unity, payable at the end of the year when death occurs, is .3675747. Proportionally, a net single premium of $16.43311 would secure $44.7341 payable at death. But $16.44311 is also the net present value at age 40 of an annual premium of one dollar. Therefore, a net level or uniform premium of $22.3543 would, at age 40, secure $1000 payable at death. [See column (6), Table No. 1.]

Let us now suppose a company to consist of 78,106 persons, each aged 40 years, each insured for $1000, or $78,106,000 in all, and each paying the net annual premium of $22.3543. The following table No. 3 has been prepared to show the progress of the fund each year until the last death claim has been paid at the age of 96 years, on the basis of the American Experience Table and four per cent interest. Column

Table No. 3.

78,106 Persons. Aged 40 Years, Insured for $1,000 Each.

Age. x	Premiums.	Fund at Beginning of Year.	Interest 4%.	Death Claims.	Fund at End of Year.	Share of Each Person in the Fund at End of Year or Net Reserve.
	(1)	(2)	(3)	(4)	(5)	(6)
40	$1,746,030	$1,746,030	$69,840	$765,000	$1,050,870	13.59
41	1,728,930	2,779,800	111,190	774,000	2,116,990	27.65
42	1,711,630	3,828,620	153,140	785,000	3,196,760	42 18
43	1,694,080	4,890,840	195,630	797,000	4,289,470	57.20
44	1,676,260	5,965,730	238,630	812,000	5,392,360	72 70
45	1,658,110	7,050,470	282,020	828,000	6,504,490	88.68
46	1,639,600	8,144,090	325,760	848,000	7,621,850	105 13
47	1 620,640	9,242,490	369,700	870,000	8,742,190	122.05
48	1,601,190	10,343,380	413,740	896,000	9,861.120	139.42
49	1,581,170	11,442,290	457,690	927,000	10 972,980	157.19
50	1.560,440	12,533,420	501,340	962,000	12,072,760	175.37
51	1,538,940	13,611,700	544,470	1 001,000	13,155,170	193.91
52	1.516,560	14,671,730	586,870	1 044 000	14.214,600	212.80
53	1,493,220	15,707,820	628,310	1,091,000	15,245,130	232.02
54	1,468,830	16,713,960	668 560	1,143.000	16,239,520	251.53
55	1 443,290	17,682,810	707,310	1,199.000	17,191,120	271.30
56	1,416,480	18,607,600	744,300	1 260,000	18,091,900	291.31
57	1,388 310	19,480,210	779 210	1,325,000	18.934,420	311.52
58	1,358,680	20,293,100	811 720	1,394,000	19.710,820	331.91
59	1,327,520	21,038,340	841 530	1,408,000	20 411,870	352.43
60	1,294,710	21,706,580	868,260	1 546,000	21,028 840	373.04
61	1,260,150	22.288,990	891,560	1,628,000	21,552,550	393.70
62	1,223,750	22,776,300	911,050	1.713,000	21.974,350	414.37
63	1,185,450	23,159,800	926,390	1,800,000	22,286,190	435.01
64	1 145,210	23,431,400	937,260	1,889 000	22,479,660	455.59
65	1.102,480	23,582,140	943,280	1,980,000	22,545,420	476.03
66	1,058,720	23,604,140	944,160	2,070.000	22,478,300	496.31
67	1,012,450	23,490,750	939,630	2,158,000	22,272,380	516.36
68	964,210	23,236,590	929.460	2,243,000	21,923.050	536 15
69	914.070	22,837.120	913.490	2 321,000	21,429,610	555.62
70	862,180	22,291,790	891 670	2,391,000	20,792,460	574.73
71	808,740	21,601,220	864,050	2,448,000	20,017,270	593.45
72	754,010	20,771,280	830,850	2,487,000	19,115,130	611 82
73	698,420	19,813,550	792,540	2,505,000	18,101,090	629.86
74	642,420	18,743,510	749.740	2,501,000	16,992.250	647 64
75	586,510	17,578,760	703.150	2,476.000	15,805,910	665.20
76	531,170	16,337,100	653,480	2,431.000	14,559,580	682 58
77	476,830	15,036,410	601,460	2,369,000	13,268,870	699.79
78	423,870	13,692,740	547,710	2,291.000	11,949,450	716.82
79	372,650	12,322,110	492,880	2,196,000	10,618,980	733.65
80	323 560	10,942,540	437,700	2,091,000	9,289,240	750 97
81	276,820	9,566.060	382,640	1 964 000	7,984.700	766.36
82	232 910	8,217,610	328,700	1 816,000	6 730,310	782 32
83	192,320	6,922,630	276,900	1,648,000	5 551,530	798.20
84	155,480	5,707,010	228.280	1.470,000	4,465,290	814 10
85	122,620	4,587,910	183.520	1,292,000	3,479.430	829 82
86	93,740	3,573.170	142,930	1.114,000	2,602,100	844 79
87	68,630	2,670,730	106,830	933,000	1,844 560	859.54
88	47,980	1,892,540	75,900	744,000	1,224 240	873 21
89	31,340	1,255,580	50,220	555,000	750,800	886.42
90	18,940	769,740	30,790	385,000	415.530	899.42
91	10,330	425,860	17 830	246,000	196,890	911.53
92	4.830	201,720	8.070	137 000	72,790	921.39
93	1,770	74,560	2.980	58,000	19,540	930 49
94	470	26,010	800	18,000	2,810	936.67
95	70	2,880	120	3,000	.....	1000.00

(1) shows the total premiums paid by those alive at the beginning of each successive year. Column (2) shows the fund at the beginning of each year just after the premiums have been paid. Column (3) shows the interest on the fund each year. Column (4) shows the death claims in each year. Column (5) shows the fund at the end of each successive year. Column (6) shows the share held for account of each survivor in each successive year (found by dividing the total fund by the number of persons surviving), and this is also the net investment reserve upon each policy.

The functions of the investment reserve will be made clearly apparent by a study of Table No. 4, which has been prepared to illustrate the appropriation each year of the component parts of an ordinary whole life level premium of $313, paid annually in advance, to secure $10,000 at the death of a man now aged 40 years (or, rather, at the end of the year when death occurs). Column (1) shows the net reserve at the end of each successive year. Column (2) shows the corresponding net amount at risk borne by the company during each successive year. This is always the difference between the face of the policy and the net reserve, which last, being in hand, is not subject to any insurance risks. Column (3) shows the net cost to insure $10,000 during each separate year by the scale of natural premiums, as indicated in column (5), Table 1. Column (4) shows the cost to insure the net amounts at risk at the successive ages indicated in the margins. Column (5) shows the deposit portion of the annual premium in each year, which, until the age of 68 is attained in the example given, goes to swell the investment reserve or accumulated deposit. After the age of 68 the yearly costs to insure the net amount at risk exceed the entire net premiums, and hence the deficiencies (as indicated by the minus sign) must be supplied by drawing from the reserve fund.

From the foregoing it will be apparent :

(1.) Every level premium policy is in reality a contract for a yearly decreasing amount of insurance, and a yearly increasing amount of investment. It is a combination of insurance, which is one thing, with investment, which is quite another thing. There is no necessary connection between the two. Insurance or indemnity may be purchased without investment, as investment may be purchased without insurance. The investment element does not add to the security of the insurance, the yearly cost of which depends, under any and every form of policy, upon the net amount at risk borne by the company, and the actual, present, attained age of the person whose life is exposed to mortality. For instance, in the example given (Table No. 4) of a whole life insurance policy of $10,000, issued at the age of 40, the reserve or invested deposits, at the end of twenty years, or at age 60, is $3,730.35. Now, this sum is in hand, and is not subject to any insurance hazard, hence the net amount at risk for that year is $6,269.65 only. The cost to insure $10,000 for one year at age 60, as shown in column (3), is $256.67. Proportionately the cost to insure $6,269.65, the net amount at risk, is $160.92, and this is all the insurance done by the company with respect to that policy during that year. At age 70 the net amount at risk is only $4,254.74, the cost of which for that year, $253.50, is $29.96 more than the net an-

Table No. 4.

Whole Life Insurance by Level or Uniform Premiums. Age at Issue 40 Years. Amount Insured $10,000. Annual Premium During Life, $313.

Age.	Net Reserve or Accumulated Deposits, being *Sel/*-Insurance at End of Year.	Net Amount of Insurance Carried by the Company During the Year.	Tabular Cost to Insure $10,000 During Each Year. Am. Exp. Table 4 per cent.	Diff'n to Insure the Net Amount at risk Each Year, being also the Full *Insurance* Reserve each Year.	Deposit Portion of each Premium which is merely for Accumulation.	Expense Portion of Each Year's Premium.	Total Yearly Premium as per Terms of the Policy Contract.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
40	$135.83	$9,864.12	$94.18	$92 90	$130 64	$89.46	$313.00
41	276 49	9,723.51	96.23	93.59	129.95	89.46	313.00
42	421 83	9,578.17	98.58	94 42	129.12	89 46	313.00
43	572.04	9,427.96	101.13	95 34	128 20	89 46	313.00
44	726.98	9,273.02	104.12	96 55	126 99	89.46	313.00
45	886 82	9,113.08	107 34	97.82	125 72	89.46	313 60
46	1,051.31	8,948 69	111.17	99.48	124.06	89.46	313.00
47	1,220.50	8,779.50	115 39	101.31	122 23	89.46	313 00
48	1,394.15	8,605.85	120.28	103.51	120.03	89.46	313 00
49	1,571.94	8,428.06	126.02	106.21	117 33	89.46	313 00
50	1,753.66	8,246 34	132 51	109.27	114 27	89.46	313 00
51	1,939 08	8,060.92	139.81	112.70	110.84	89.46	313.00
52	2,127.99	7,872.01	147.97	116.48	107.06	89 46	313.00
53	2,320.16	7,679.84	157 05	120.61	102.93	89.46	313 00
54	2,515.25	7,484 75	167.27	125 20	98.34	89 46	313.00
55	2,713.02	7,286.98	178 57	130.12	93 42	89 46	313.00
56	2,913.10	7,086.90	191.20	135.50	88 04	89.46	313.00
57	3.115.22	6 884.78	205.15	141.24	8 .30	89 46	313.00
58	3,319.09	6,680.91	220.03	147.00	76.54	89.46	313.00
59	3,524 25	6,475.75	237 69	153.93	69 61	89.46	313.00
60	3,730.35	6,269.65	256.67	160 92	62.62	89.46	313 00
61	3,936.95	6,063.05	277.69	169 35	54 19	89.46	313.00
62	4,143.66	5,856.34	300 88	176 20	47 34	89 46	313.00
63	4.350.12	5.649 88	318.95	180.20	43.34	89.46	313 00
64	4 555 86	5,444.14	354.54	193.01	30.53	89.46	313 00
65	4,760 33	5,239.67	385.85	202.18	21 36	89.46	313 00
66	4,963 07	5,036.93	420,26	211 68	11.86	89.46	313.00
67	5,163 64	4.836.36	458.15	221.58	1.96	89.46	313.00
68	5,361.46	4,638.54	500.02	231.94	—8.40	89.46	313.00
69	5,556.16	4,443.84	545.79	242 53	—18.99	89.46	313 00
70	5,747 26	4,252.74	596.08	253 50	—29.96	89.46	313 00
71	5,934.54	4,065 46	650 63	264 61	—41.07	89 46	313.00
72	6,118 19	3.881.81	708 97	275.23	—51.69	89.46	313.00
73	6,298.64	3,701.36	770 94	285.35	—61 81	89.46	313.00
74	6,476.42	3,523 58	836.80	294.85	—71.31	89 46	313.00
75	6,652.02	3,347 98	907.41	303.80	—80.26	89 46	313 00
76	6.825.83	3,174 17	983 76	312.26	—88 72	89.46	313.00
77	6,997 93	3,002.07	1,067.93	320.60	—97.06	89 46	313.00
78	7,168.17	2,831.83	1,161.80	329.00	—105.46	89.46	313.00
79	7,336.51	2,663.49	1,266.67	337.22	—113 68	89.46	313 00
80	7,509.70	2,490 30	1,389 10	345.90	—122.36	89.46	313 00
81	7,663 60	2.336.40	1,525.04	356.31	—132.77	89.46	313.00
82	7,823.20	2,176.80	1,675.93	364.83	—141 29	89.46	313.00
83	7,982 00	2,018.00	1,841.93	371 70	—149 16	89.46	313.00
84	8,141 00	1,859 00	2,032.30	377 81	—154.27	89.46	313.00
85	8 298,20	1,701.80	2,264 92	385.44	—161 90	89.46	313.00
86	8,447.90	1,552.40	2,554.63	396.57	—173.03	89.46	313.00
87	8,595.40	1,404.60	2,913.66	409 26	—185.72	89.46	313.00
88	8,732.10	1,267.90	3,335.57	422 61	—199.07	89.46	313.00
89	8,864.20	1,135.80	3,806.38	432.32	—208.78	89.46	313.00
90	8,994.20	1,005 80	4,370 63	439 60	—216.06	89 46	313.00
91	9,115.30	884.70	5,119 88	452.96	—229.42	89.46	313.00
92	9,213.90	786.10	6,098.68	479.42	—255 88	89.46	313 00
93	9,304.90	695,10	7,059.40	490.69	—267.15	89 46	313 00
94	9,366,70	633.30	8,241.76	521.96	—298.42	89 46	313 00
95	10,000.00	.....	9,615.40	.....	.....	89.46	313.00

nual premium ($223.54). The deficiency for that year, as well as the deficiencies for each subsequent year, as shown in column (5), must be met by drawing on the investment reserve, or accumulated fund, the express functions of which is to provide for the excessive cost of insurance in old age when the level premium is insufficient for that purpose.

(2). The investment reserve is occasioned solely by the artificial condition in the level premium contract, which provides that the premiums shall not increase as the insured grows older, and to enable the company to pay the sum insured as an endowment.

(3). Whether the combination of insurance and investment is desirable or advantageous, depends upon the manner in which each is administered. If either the insurance or the investment can be obtained on better terms separately, the combination of the two is certainly undesirable and disadvantageous to the policyholder.

Instead of contracting with a life insurance company for both insurance and investment, which together make up the sum insured, two separate contracts might be made—the one with a life company for the yearly decreasing amounts of insurance only, see column (2) table 4, the other with a savings bank or trust company for accumulating the deposit, or investment portions of the yearly premium, see column (5) of the same table. In case of death in such case the insurance company would pay the net amount insured only, column (2), while the savings bank would pay the accumulated deposits, column (1), the two together making up the full amount guaranteed.

To show even more clearly how the insurance and investment elements may be completely separated the following tables have been prepared.

Table No. 5 illustrates the case of an endowment assurance issued at age of forty years for $10,000 payable in ten years or at death if prior. The net premium only ($853.62) is considered—the margin for expenses and adverse contingencies being disregarded.

Tables 6 and 7 are intended to show how the same result can be secured by purchasing a ten-year term insurance with the insurance company, annual premium $106.03, and a pure endowment (payable only in case of survival) by depositing the residue ($747.59) of the endowment assurance premium for accumulation. In case of death at any time during the ten years, the insurance company would pay the full amount insured, and the endowment fund would be lost. In case of surviving, the $10,000 would be paid as an endowment, and the insurance would cease.

The same principles apply to any other term of years, as a whole life policy is in reality an endowment assurance payable on attaining the age of ninety-six years, or at death if prior.

Comparison of an endowment assurance contract, a ten year term level premium contract, and a pure endowment contract. Amount $10,000, and age at issue 40 years, in each case:

TABLE NO. 5.

ENDOWMENT ASSURANCE, ANNUAL PREMIUM $853.62.

YEAR.	Net Reserve or Accumulated Deposits Being Self-Insurance.	Net Amount of Insurance at Risk or Carried by the Company.	Tabular Cost Each Year to Insure $10,000 for the Year.	Tabular Cost to Insure Net Amount at Risk which is also the Full Legal and Mathematical Insurance Reserve.	Deposit Portion of Annual Premium Which is Merely for Accumulation.
1	$797.63	$9.202 37	$94 18	$86.67	$766 95
2	1,633 57	8,366 43	96 23	80.51	773.11
3	2,509 89	7,490.11	98.58	73.84	779.78
4	3,428.95	6,571.05	101.13	66 45	787 17
5	4,393.16	5,606.84	104.12	58.38	795.24
6	5,405.36	4,494 64	107.34	48.24	805.38
7	6,468.51	3,531.49	111.17	39.26	814.36
8	7,586.05	2,413.95	115.39	27.85	825.77
9	8,761.76	1,238.24	120.28	14.89	838.73
10	10,000.00	Nil.	126.02	Nil.	853.62

TABLE NO. 6.

TEN-YEAR TERM INSURANCE, NET ANNUAL PREMIUM $106.03.

YEAR.	Net Reserve or Accumulated Deposits Being Self-Insurance.	Net Amount of Insurance at Risk or Carried by the Company.	Tabular Cost Each Year to Insure $10,000 for the Year.	Tabular Cost to Insure Net Amount at Risk which is also the Full Legal and Mathematical Insurance Reserve.	Deposit Portion of Annual Premium Which is Merely for Accumulation.
1	$12.45	$9,987.55	$94.18	$94.06	$11.97
2	23.37	9,976.63	96.23	96.00	10.03
3	32.37	9,967.63	98 58	98.26	7.77
4	39.18	9,960.82	101.13	100.73	5.30
5	43.20	9,956.80	104.12	103.57	2.37
6	44.05	9,955.95	107.34	106.87	—0.84
7	40.95	9,959.05	111.17	110.72	—4.69
8	33.24	9,906.76	115.39	115.01	—8.98
9	19.99	9,980 01	120.28	120.04	—14.01
10	.....	10,000.00	126.02	126 02	—19.99

TABLE NO. 7.

PURE ENDOWMENT—AGE 40 AT ISSUE—$10,000 PAYABLE ONLY IN CASE OF BEING ALIVE AT THE END OF 10 YEARS, OR AT AGE 50.

YEAR.	Yearly Payments.	Value (Without Interest) of $1.00 Payable Only in Case of Surviving to End of Year.	Fund at Beginning of Year.	Value of Ditto Payable Only in Case of Survivir g.	Interest 4%.	Fund at End of Year.
1	$747.59	$1,009.89	$747.59	$754 99	$30.20	$785.19
2	747.59	1,010.11	1,532.78	1,548.28	61.93	1,610.21
3	747.59	1,010 36	2,357.80	2,382.23	95.29	2,477.51
4	747.59	1,010.63	3,225.10	3,259 38	130.37	3,389 75
5	747.59	1,010.95	4,137.34	4,182.63	167.31	4,349.94
6	747.59	1,011.29	5,097.53	5,155.08	206.20	5,361.28
7	747.59	1,011.70	6,108.87	6,180.34	247.21	6,427.55
8	747.59	1,012.15	7,175.14	7,262.32	290.49	7,552.81
9	747.59	1,012.67	8,300.40	8,405 56	336.22	8,741.78
10	747.59	1,013 28	9,489.37	9,615.39	384.61	10.000.00

Insurance and investment therefore have no necessary connection—either one may be obtained without the other.

(4). Pure insurance, unmixed with banking or investment, involves the payment of natural premiums, which inevitably and inexorably increase with age. The only way to avoid these increasing rates is to pay largely in excess of the requirements for current death claims in the earlier years, and thus provide a fund upon which to draw in the later years—that is to say, by combining investment with insurance. The first is known as the natural premium plan, the second as the level premium plan. Properly administered, the one is as safe and as sound as the other, as both depend upon the application of the same laws of nature which govern the rates of mortality, or the probability of living and dying in each successive year of life. In fact, as before stated, level premiums are simply the commuted equivalents of the increasing or natural premiums. In both systems, the company must alike be furnished with the cost of insuring the net amount at risk at the actual age attained on each and every policy in force. This cost is independent of the form of policy contract, the age at issue, or the scale of premium charged. This cost, as previously stated, may be furnished either by direct, present payments, as by natural premiums, or partly by direct present payments, and partly by drawing upon the investment reserve or accumulated deposits, a fund contributed by the policyholder for this express purpose.

There are only two sound systems of life insurance; the one by natural premiums, increasing each year as a man grows older; the other, by level premiums, which necessitate investments or accumulated payments largely in excess during the earlier years to meet the deficiencies of the uniform, unchanging premiums in later years. The attempts by so many co-operative or assessment companies to furnish insurance by assessments based upon the age at entry, and which rates do not increase with age must inevitably result in disappointment and disaster. Natural laws may not be violated with impunity.

SHEPPARD HOMANS.

NEW YORK, May 10, 1888.